WEAK DISCONTINUITIES IN AN ELECTRICALLY CONDUCTING MAGNETIZED LIQUID

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UDC 538.3:538.4

1. We will discuss the propagation of weak discontinuities in a conducting magnetized liquid. The liquid is assumed to be incompressible (ρ = const), ideal (dissipative forces are absent), and nonuniformly and isotropically magnetized according to the arbitrary law $\mu = \mu(\rho, T, H)$, so that the magnetic induction B and the strength of the magnetic field H are related by the equation

$$\mathbf{B} = \mu(\rho, T, H)\mathbf{H}. \tag{1.1}$$

The dependence of the magnetic permeability μ on the density ρ and temperature T permits taking into account magnetostrictive and magnetocaloric effects in a magnetized liquid. Equation (1.1) has been adopted in connection with the investigation of the flows of liquid dia- and paramagnetic metals [1] and electrically conducting ferromagnetic liquids [2], in which it is possible to neglect the phenomena associated with hysteresis of magnetization.

The system of equations which describes the nonsteady motions of such a liquid in the magnetohydrodynamic approximation is of the form [3]

$$\operatorname{div}\mathbf{v} = 0, \quad \operatorname{div}\mathbf{B} = 0, \ \frac{d}{dt}(S + S^e) = 0, \tag{1.2}$$

$$\rho\frac{d\mathbf{v}}{dt} = -\nabla(p + \psi) + \frac{1}{4\pi}\operatorname{rot}\mathbf{H}\times\mathbf{B} + M\nabla H, \quad \frac{\partial\mathbf{B}}{\partial t} = \operatorname{rot}[\mathbf{v}\times\mathbf{B}].$$

Here $\mathbf{v}$ is the fluid velocity, $M = (\mu - 1)H/(4\pi)$ is the magnetization intensity,

$$\psi = \int_0^H (\mu - 1 - \rho\mu_\rho)HdH, \quad S^e = \frac{1}{4\pi\rho}\int_0^H \mu_T HdH, \tag{1.3}$$

and u_ρ, u_T... denote the partial derivatives of the function u with respect to ρ and T, respectively, with the other parameters constant.

The entropy S and temperature T of the liquid satisfy the equation T = T(S) [4] in the absence of an electromagnetic field.

The discontinuity of the first derivatives of the arbitrary function u upon passing through the surface $\varphi(x, y, z, t) = 0$ of a weak discontinuity is determined by a single function λ_u [5], so that

$$\langle\nabla u\rangle = \lambda_u\mathbf{n}, \ \langle\partial u/\partial t\rangle = -\lambda_u G, \tag{1.4}$$

where $\mathbf{n}$ is the unit normal vector to the surface $\varphi(x, y, z, t) = 0$ and $G = -\mathbf{n}\frac{\partial\varphi}{\partial t}\Big/\sqrt{|\nabla\varphi|^2}$ is the propagation velocity of the discontinuity; the angular brackets denote a discontinuity of the quantity contained within them upon passing through the discontinuity surface.

Using (1.4), we obtain from (1.2) the dynamic conditions at weak discontinuities

$$(\lambda_v\mathbf{n}) = 0, \quad (\lambda_B\mathbf{n}) = 0, \quad \theta(\lambda_S + \lambda_{S^e}) = 0, \tag{1.5}$$

$$\rho\theta\lambda_v = (\lambda_p + \lambda_\psi)\mathbf{n} + \frac{\mathbf{B}}{4\pi}\times(\mathbf{n}\times\lambda_H) - M\lambda_H\mathbf{n},$$

$$\lambda_B\theta + \lambda_v B_n = 0,$$

where $\theta = G - v_n$ is the normal component of the velocity of an element of the discontinuity with respect to the medium, $v_n = (\mathbf{vn})$, $B_n = (\mathbf{Bn})$, $\lambda_v = \lambda_{v_x}\mathbf{i} + \lambda_{v_y}\mathbf{j} + \lambda_{v_z}\mathbf{k}$, and $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ are the unit vectors of the cartesian coordinate system x, y, and z.

We have in addition

Kharkov. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 2, pp. 28-37, March-April, 1980. Original article submitted February 14, 1979.

0021-8944/80/2102-0182 $07.50

$$\lambda_T = T_S\lambda_S,\ \lambda_\psi = \psi_T\lambda_T + (\mu - 1 - \rho\mu_\rho)H\lambda_H/(4\pi),$$
$$\lambda_{Se} = S_T^e\lambda_T + \mu_T H\lambda_H/(4\pi\rho),$$
$$\lambda_H = (\mu(\boldsymbol{\lambda}_B\mathbf{B})/B - \mu_T\lambda_T B)/(\mu^2 + \mu_H B),$$
$$\boldsymbol{\lambda}_H = \frac{\boldsymbol{\lambda}_B}{\mu} - \frac{\mu_H(\boldsymbol{\lambda}_B\mathbf{B})\,\mathbf{B}}{\mu B\left(\mu^2 + \mu_H B\right)} - \frac{\mu_T\lambda_T\mathbf{B}}{\mu^2 + \mu_H B} \tag{1.6}$$

from Eqs. (1.1), (1.3), and the equation of state.

We will represent each of the vectors in the form $\mathbf{v} = \mathbf{v}_\tau + v_n\mathbf{n}$, where $\mathbf{v}_\tau$ is the projection of the vector $\mathbf{v}$ onto the plane tangent to the discontinuity surface. Taking account of the fact that $\lambda_{v_\tau} = \lambda_v - \lambda_{v_n}\mathbf{n}$ and using Eqs. (1.6), we transform the system of equations (1.5) to the form

$$\lambda_{v_n} = 0,\quad \lambda_{B_n} = 0; \tag{1.7}$$
$$\lambda_p = -\lambda_\psi - (\mathbf{B}\boldsymbol{\lambda}_H)/(4\pi) + B_n(\boldsymbol{\lambda}_H\,\mathbf{n})/(4\pi) + M\lambda_H; \tag{1.8}$$
$$B_n\lambda_{v_\tau} = -\theta\lambda_{B_\tau}; \tag{1.9}$$
$$\rho\theta\lambda_{v_\tau} = -\frac{B_n}{4\pi}\left\{\frac{\lambda_{B_\tau}}{\mu} - \frac{\mu_H(\boldsymbol{\lambda}_{B_\tau}\mathbf{B}_\tau)}{\mu^2 + \mu_H B}\frac{\mathbf{B}_\tau}{\mu B} - \frac{\mu_T T_S\lambda_S\mathbf{B}_\tau}{\mu^2 + \mu_H B}\right\}; \tag{1.10}$$
$$\theta\left[\lambda_S + N\mu\mu_T m\left(\boldsymbol{\lambda}_{B_\tau}\mathbf{B}_\tau\right)\right] = 0,$$
$$m = \left[4\pi\rho\mu\left(\mu^2 + \mu_H B\right)\right]^{-1},\qquad N = \left[1 + S_T^e T_S - \mu_T^2 T_S m B^2\right]^{-1}. \tag{1.11}$$

It follows from Eqs. (1.7) that the derivatives of the normal components of the velocity and magnetic field induction are continuous on the characteristic, so that the discontinuity can undergo only derivatives of the tangential components of the velocity and induction. Equation (1.8) determines the intensity of the discontinuity of the pressure derivatives. The system of homogeneous equations (1.9)-(1.11) serves to determine λ_S, λ_{v_τ}, ν_{B_τ}. We obtain from the existence condition of nontrivial solutions of this system an equation which determines the possible velocities θ of weak discontinuities in a conducting magnetized liquid:

$$\theta\left(\theta^2 - \frac{B_n^2}{4\pi\rho\mu}\right)\left[\theta^2 - mB_n^2\left(\mu^2 + \mu_H B_n^2/B\right) - N\mu^2\mu_T^2 T_S m^2 B_n^2 B_\tau^2\right] = 0. \tag{1.12}$$

Thus the following types of weak discontinuities exist in an electrically conducting, incompressible, magnetized liquid:

magnetohydrodynamic (Alfvén)

$$\theta_A^2 = mB_n^2\left(\mu^2 + \mu_H B\right); \tag{1.13}$$

magnetosonic

$$\theta_M^2 = mB_n^2\left[\mu^2 + \mu_H B_n^2/B + N\mu^2\mu_T^2 T_S m B_\tau^2\right]; \tag{1.14}$$

and entropic

$$\theta_S = 0. \tag{1.15}$$

Thus magnetostrictive effects in an incompressible liquid affect only the intensity of the pressure discontinuity. If $\mu = \mu(\rho) = \text{const}$, only Alfvén and entropic discontinuities are possible in an electrically conducting liquid, so that a magnetosonic discontinuity is exhibited in an isotropically magnetized liquid exclusively due to nonuniformity of the magnetization law.

The possibility of plane magnetosonic waves of infinitely small amplitude in a conducting magnetized liquid was first pointed out in [6].

We obtain from (1.13) and (1.14) that the propagation velocity of a magnetosonic discontinuity is equal to the Alfvén velocity only in the cases in which the magnetic induction vector is orthogonal to a discontinuity element ($B_\tau = 0$), tangent to it ($B_n = 0$), or for parameter values for which

$$\mu^2\mu_T^2 T_S m B = \mu_H\left[1 + S_T^e T_S - \mu_T^2 T_S m B^2\right]. \tag{1.16}$$

In the latter case the magnetosonic velocity coincides with the Alfvén velocity for an arbitrary orientation of the discontinuity elements relative to the field. We note that this case is impossible if the dependence of the magnetic permeability only on the temperature or only on the magnetic field intensity is taken into account.

We will determine the quantities λ_i ($i = \rho$, T, ...) in each type of discontinuity. From (1.9) and (1.10) we have

$$\left[\theta^2-(\mu^2+\mu_H B_n^2/B)\, mB_n^2\right](\lambda_{B_\tau}\mathbf{B}_\tau)=-\mu\mu_\tau T_S m B_n^2 B_\tau^2 \lambda_S,$$

and Eq. (1.11) is reduced to the form

$$\theta(\theta^2-\theta_M^2)\left[\theta^2-mB_n^2(\mu^2+\mu_H B_n^2/B)\right]^{-1}\lambda_S=0.$$

From this $\lambda_S = 0$ follows for a magnetohydrodynamic discontinuity ($\theta = \theta_A$), so that $\lambda_T = 0$. From (1.6)-(1.9) we have

$$\lambda_{v_n}=\lambda_{B_n}=\lambda_H=\lambda_{S^e}=\lambda_p=(\lambda_v\mathbf{B})=(\lambda_B\mathbf{B})=0,$$

$$\lambda_H=\lambda_B/\mu,\quad \lambda_{v_\tau}=\pm\frac{1}{\sqrt{4\pi\rho\mu}}\lambda_{B_\tau}.$$

Only the tangential components of the velocity, induction, and magnetic field intensity are altered in magnetohydrodynamic discontinuities.

The properties of magnetosonic discontinuities depend significantly on taking the magnetocaloric effect in liquids into account. First we will consider the case in which $\mu_T = 0$. Such magnetization laws occur in particular in diamagnetic liquids and in a paramagnetic liquid in strong fields when the magnetization is close to saturation.

For this case $S^e = 0$, and we obtain $\lambda_S = 0$ from (1.11) and

$$\lambda_T=0,\quad \lambda_p=-(\lambda_{B_\tau}\mathbf{B}_\tau)\, m\mu\rho\left[\mu-\rho\mu_\rho+\mu_H B_n^2/(\mu B)\right],$$

$$\lambda_{B_\tau}=(\lambda_{B_\tau}\mathbf{B}_\tau)\frac{\mathbf{B}_\tau}{B_\tau^2},\tag{1.17}$$

$$\lambda_{v_\tau}=-\frac{\theta}{B_n}\lambda_{B_\tau},\quad \lambda_H=4\pi\rho\mu^2 m(\lambda_{B_\tau}\mathbf{B}_\tau)/B$$

from (1.6-(1.8).

Consequently, the derivatives of thermodynamic parameters are continuous.

For the case $\mu_T \neq 0$ we obtain from (1.6)-(1.11)

$$\lambda_T=T_S\lambda_S,\quad \lambda_H=-\lambda_S(1+S_T^e T_S)/(\mu^2+\mu_H B)\,\mu_T mB,\tag{1.18}$$

$$\lambda_p=\frac{\lambda_S}{\mu\mu_T}\left\{-\psi_T T_S\mu\mu_T+\rho(1+S_T^e T_S)\left[(2-\rho\mu_\rho)\mu+\mu_H B_n^2/B\right]-\rho\mu_T^2 T_S m B_n^2(\mu^2+\mu_H B)\right\},$$

$$\lambda_{B_\tau}=-\lambda_S\mathbf{B}_\tau/N\mu\mu_T m B_\tau^2,\quad \lambda_{v_\tau}=-\frac{B}{B_n}\lambda_{B_\tau}.$$

The derivatives of thermodynamic parameters undergo discontinuities in connection with taking the magnetocaloric effect in weak discontinuities into account. It follows from (1.17) and (1.18) that the discontinuities of the derivatives of the tangential components of the velocity and induction lie in a plane which passes through **n** and B_T. Thus in contrast to the Alfvén discontinuities the magnetosonic discontinuities are plane polarized.

Now let an element of a magnetosonic discontinuity move with the Alfvén velocity. Then if $B_T = 0$, we obtain the same relationships for the discontinuities of the derivatives as for magnetohydrodynamic discontinuities. If $B_n = 0$ or if condition (1.16) is satisfied, the discontinuities of the derivatives of the magnetohydrodynamic parameters are arbitrary in general.

It follows from Eq. (1.9) for an entropic discontinuity ($\theta = 0$) that $\lambda_{v_T} B_n = 0$. If $B_n \neq 0$, then $\lambda_{v_T} = 0$. Thereby we have from (1.10)

$$\lambda_{B_\tau}=\mu\mu_T T_S B\mathbf{B}_\tau\lambda_S/(\mu^2 B+\mu_H B_n^2),$$

$$\lambda_H=-\mu_T T_S B_n^2\lambda_S/(\mu^2 B+\mu_H B_n^2),$$

$$\lambda_p=-\left[\psi_T+\frac{\rho\mu_\rho\mu_T B_n^2 B}{4\pi\mu(\mu^2 B+\mu_H B_n^2)}\right]T_S\lambda_S.$$

In this case the derivatives of the velocity on the entropic discontinuity are continuous. If $\mu_T = 0$, only the temperature and entropy undergo a discontinuity at an entropic discontinuity.

If $B_n = 0$, then λ_{v_T}, λ_{B_T} and λ_S are arbitrary, and λ_p and λ_H are determined from Eqs. (1.6) and (1.8).

2. Let us investigate the question of the propagation of weak discontinuities in a conducting magnetized liquid.

The magnetization M of paramagnetic liquids is determined by Langevin's formula [1]:

$$M = M_0 L(\xi),\ L(\xi) = \operatorname{cth}\xi - \xi^{-1},\ \xi = m_0 H/kT, \tag{2.1}$$

where m_0 is the magnetic moment of the molecule, k is Boltzmann's constant, and M_0 = const.

Equation (2.1) is adopted for ferromagnetic liquids [7]; in this case m_0 is the magnetic moment of single-domain particles of a dispersed ferromagnet.

Equation (1.16) has the form

$$f(\xi) = \zeta(\xi \operatorname{cth}\xi - 1)(1 - \xi^2/\operatorname{sh}^2\xi) + \xi \operatorname{cth}\xi + \xi^2/\operatorname{sh}^2\xi - 2 = 0,$$
$$\zeta = M_0 k T_S / m_0 \rho T$$

in the case of Langevin's law.

For values $0 < \xi < \infty$ this equation has no solutions, and the function $f(\xi)$ is positive. Thus $\theta_M^2 \geq \theta_A^2$ in a paramagnetic liquid.

For the case $\mu = \mu(H)$ we have from (1.12)

$$\frac{\theta_M^2}{\theta_A^2} = 1 - \frac{\mu_H B_\tau^2/B}{\mu^2 + \mu_H B}. \tag{2.2}$$

The known magnetization laws, as well as the kinetic representations on the nature of the magnetization of dia- and paramagnetic substances [1] permit assuming that for the magnetization M(H) in a paramagnetic substance $dM/dH > 0$ and $d^2M/dH^2 \leq 0$ and in a diamagnetic substance $dM/dH < 0$ and $d^2M/dH^2 \geq 0$. Then we have for the magnetic permeability [8]

$$0 \leqslant \mu_H \leqslant (1 - \mu)/H (\mu < 1),\ (1 - \mu)/H \leqslant \mu_H \leqslant 0\ (\mu > 1). \tag{2.3}$$

Therefore it follows from (2.2) that in a paramagnetic liquid the velocity of magnetosonic discontinuities is greater than the Alfvén velocity, and in diamagnetic liquids it is always less (with the exception of discontinuity elements for which $B_n = 0$ or $B_\tau = 0$).

We will represent the velocity of magnetosonic discontinuities in the form

$$\theta^2 = al + bl(1 - l),\ a = B^2/4\pi\rho\mu,\ l = \cos^2\varphi,\ \cos\varphi = B_n/B. \tag{2.4}$$

In a paramagnetic liquid

$$b = mB^2\left(N\mu^2\mu_T^2 T_S m B^2 - \mu_H B\right),$$

and one can assume $b > 0$ by virtue of what has been set forth above.

In a diamagnetic liquid

$$b = -\frac{\mu_H B^3}{4\pi\rho\mu\left(\mu^2 + \mu_H B\right)} < 0,$$

with $a + b > 0$.

The general appearance of the phase-velocity diagrams is illustrated in Figs. 1 and 2, where l_A and l_M denote the $\theta(\varphi)$ curves for magnetohydrodynamic and magnetosonic discontinuities, respectively (one-fourth of the diagrams are given). The phase-velocity diagrams in a liquid magnetized according to an arbitrary law $\mu = \mu(\rho, T, H)$ have the same appearance if one excludes from the discussion the degenerate case, in which Eq. (1.16) is satisfied. In the latter case the curves l_A and l_M coincide.

Let the state of the liquid be characterized by the parameter values

$$\mathbf{v}(\mathbf{r}, t) = 0,\ \mathbf{B}(\mathbf{r}, t) = \text{const},\ T(\mathbf{r}, t) = \text{const},\ p(\mathbf{r}, t) = \text{const},$$
$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}.$$

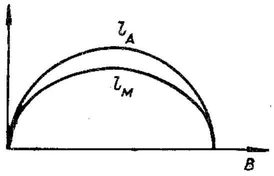

Fig. 1

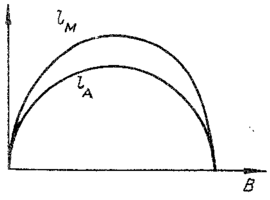

Fig. 2

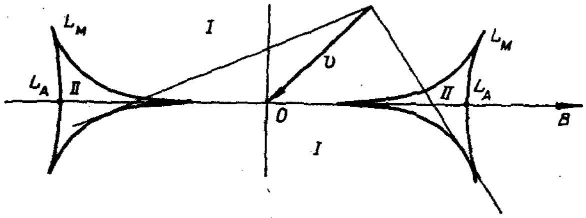

Fig. 3

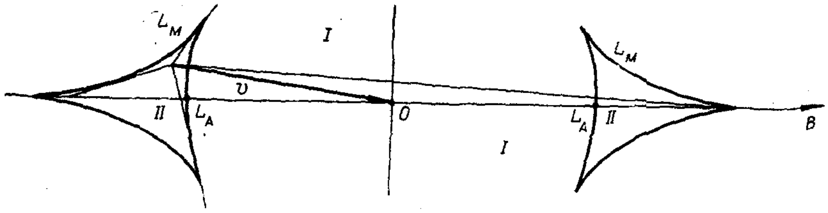

Fig. 4

Using the results of Ref. 9, we obtain that each element of the discontinuity surface moves through the liquid at constant velocity without rotating. The velocity component normal to the discontinuity element coincides with one of the velocities θ (see (1.12)). If the surface of a weak discontinuity is specified at the initial instant of time t_0, then at time $t = t_0 + 1$ it is an enveloping plane of the tangents to the initial discontinuity surface Ω_0 shifted by a distance θ in the direction of the normals to Ω_0.

The intersection of the discontinuity surface with the plane passing through B at time t_0+1 is illustrated in Figs. 3 and 4 for an initial discontinuity surface Ω_0 concentrated near the origin of coordinates and containing every kind of oriented elements. The L_M curves denote the cross sections of the surfaces of magnetosonic discontinuities. The points L_A denote Alfvén discontinuities. The entropic discontinuity coincides with the current surface and is illustrated by the point 0 in the diagrams.

The surfaces of weak discontinuities are specified in this case by the parametric equations [10]

$$X = \left[\theta(l) + 2\frac{d\theta}{dl}(1-l)\right]\sqrt{l}, \quad Y = \left[\theta(l) - \frac{2d\theta}{dl}\, l\right]\sqrt{1-l} \tag{2.5}$$

(the origin of the cartesian coordinate system (X, Y) coincides with the discontinuity surface at the initial instant of time, and the X axis is directed along the vector B). With the use of (2.4), Eqs. (2.5) are reduced to the form

$$X = \frac{a + b(1-l)^2}{\sqrt{a + b(1-l)}}, \quad Y = \frac{bl^{3/2}\sqrt{1-l}}{\sqrt{a + b(1-l)}} \tag{2.6}$$

for magnetosonic discontinuities.

Elementary investigations show that the curves (2.6) depict two curvilinear triangles whose tips are cuspidal points, and the convexity of these triangles is directed towards the inside of the region bounded by them. In a paramagnetic liquid these triangles are directed with the taper away from the origin of coordinates (Fig. 4) and in the diamagnetic case – towards the origin of coordinates (Fig. 3). The tips of the triangles are determined by the equations

$$l = 0, \ l = (3b + 2a - \sqrt{4a^2 + 3ab})/3b.$$

The L_M curves exist in a nonuniformly magnetized liquid for values of the magnetic permeability which differ as little as desired from unity, and they degenerate into the points L_A only in uniformly magnetized liquids. We note that the diagrams in Fig. 3 are similar to the diagrams for slow magnetosonic waves in magnetohydrodynamics. The diagrams given in Fig. 4 have no analogy in isotropic magnetohydrodynamics.

Figures 3 and 4 correspond to cases in which the dependences illustrated in Figs. 1 and 2 occur between the velocities of the discontinuities.

3. We will discuss two-dimensional steady-state motions of an ideal conducting magnetized liquid with $v_z = 0$ and $B_z = 0$. In this case the characteristic surfaces of the system of equations (1.2) are cylindrical surfaces whose generatrices are parallel to the z axis. Only magnetosonic and entropic characteristics are possible in two-dimensional motions of the liquid. These characteristics coincide with the current lines. For steady-

state motions $\theta = -v_n$. Taking this into account, we obtain from (1.14) an equation which determines the possible characteristic directions in the (x, y) plane:

$$(y')^4 [Q_x^2 - q_x^2 (R_1 + q_x^2 R_2)] - 2 (y')^3 [Q_x Q_y - q_x q_y (R_1 + 2q_x^2 R_2)]$$
$$+ (y')^2 [Q^2 - R_1 - 6q_x^2 q_y^2 R_2] - 2y' [Q_x Q_y - q_x q_y (R_1 + 2q_y^2 R_2)] \quad (3.1)$$
$$+ Q_y^2 - q_y^2 (R_1 + q_y^2 R_2) = 0,$$

where

$$y' = \frac{dy}{dx}; \quad \mathbf{Q} = \mathbf{v}/A; \quad \mathbf{q} = \mathbf{A}/A; \quad \mathbf{A} = \mathbf{B}/\sqrt{4\pi\rho\mu}; \; Q^2 = Q_x^2 + Q_y^2;$$

$$R_1 = (\mu^2 + N\mu^2\mu_T^2 T_S m B^2)/(\mu^2 + \mu_H B); \quad R_2 = (\mu_H B - N\mu^2\mu_T^2 T_S m B^2)/(\mu^2 + \mu_H B).$$

In the general case Eq. (3.1) determines either four or two characteristic directions at the flow point **r**, depending upon the magnitude of the vector **v**. We construct the L_M curves in order to determine the number and directions of the characteristics from the flow parameters at the point **r**. Then one can show similarly to Ref. 9 that the directions of the characteristics at this point coincide with the directions of the tangents drawn to the L_M curves from the point $-\mathbf{v}(\mathbf{r})$. It follows from this that if the roots of the vector $-\mathbf{v}(\mathbf{r})$ fall into the regions II bounded by the curvilinear triangles in Fig. 3 and 4, then four real characteristic directions exist at the flow point **r**. If the end of the vector $-\mathbf{v}(\mathbf{r})$ falls into the regions I, then we only have two real characteristics.

There are no characteristics in region II for the flow of a liquid in a perpendicular magnetic field, and we have two characteristics in region I. If the vectors **v** and B are parallel, then two real characteristics besides the current lines exist in region II, and there are no real magnetosonic characteristics in region I. The condition for the existence of characteristics is described in the form

$$\mu^2 + N\mu^2\mu_T^2 T_S m B^2 \leqslant v^2/mB^2 \leqslant \mu^2 + \mu_H B \; (\theta_M^2 < \theta_A^2)$$

or

$$\mu^2 + \mu_H B \leqslant v^2/mB^2 \leqslant \mu^2 + N\mu^2\mu_T^2 T_S m B^2 \; (\theta_M^2 > \theta_A^2)$$

for the flow of a liquid along the force lines of the magnetic field.

4. The presence of real magnetosonic characteristics results in significant deviations in the motion of an incompressible nonuniformly magnetized liquid in comparison with the flow of an electrically conducting liquid, for which the magnetic permeability is assumed to be constant.

We will discuss one-dimensional simple waves, which are a particular case of transverse waves, which were investigated in Ref. 11.

For an arbitrary magnetization law $\mu = \mu(\rho, T, H)$ one can derive from the system (1.2) the equations of simple waves [12] in the form

$$\frac{dT}{dB_y} = -N\mu\mu_T T_S m B_y, \quad \frac{dv_y}{dB_y} = -\theta/B_x, \quad (4.1)$$
$$B_x = \text{const}, \; v_x = \text{const}, \; B_z \equiv 0, \; v_z \equiv 0,$$

where θ is the velocity of the front of a simple wave defined by Eq. (1.14), in which it is necessary to set $B_n = B_x$.

Since the magnetization for a paramagnetic liquid decreases as the temperature increases ($\mu_T < 0$), then the temperature increases monotonically in magnetosonic simple waves, as follows from (4.1), and, on the contrary, the velocity decreases monotonically as the magnetic induction increases.

The equation

$$x - (v_x + \theta(B_y))t = F(B_y) \quad (4.2)$$

determines after integration of the system (4.1) the dependence $B_y(x, t)$ in a simple wave propagating in the positive direction of the x axis according to a specified initial magnetic induction distribution: $B_y = F^{-1}(x)$.

We obtain from (2.1) in weak magnetic fields ($\xi \ll 1$)

$$\frac{\mu - 1}{\mu} = \frac{\mu_0 - 1}{\mu_0}\frac{T_0}{T}$$

for an electrically conducting paramagnetic liquid magnetized according to Langevin's law. Taking the equation of state for the liquid in the form [4] $S = c_p \ln T + \text{const}$ (c_p is the specific heat) and neglecting terms in (4.1) of the order of $(\mu_0 - 1)^2$ and higher, we obtain after integration

$$T^2 - T_0^2 = (\mu_0 - 1)\frac{T_0}{c_p}\frac{B_y^2 - B_{y0}^2}{4\pi\rho},$$

$$v_y - v_{y0} = -\frac{B_y - B_{y0}}{\sqrt{4\pi\rho}} + \frac{(\mu_0 - 1)(B_y - B_{y0})}{2\sqrt{4\pi\rho}}. \quad (4.3)$$

With $\mu_0 = 1$ Eqs. (4.3) determine the variation of the variables in an Alfvén simple wave.

In strong magnetic fields ($\xi \gg 1$, a state of saturation of the magnetization) the magnetization M is constant, so that $\mu - 1 = 4\pi M/H$. Then we obtain from (4.1)

$$\frac{dv_y}{dB} = -\frac{1}{\sqrt{4\pi\rho}}\left(\frac{B^3 - 4\pi M B_x^2}{B^3 - BB_x^2}\right)^{1/2}, \qquad T = \text{const.} \quad (4.4)$$

Restricting ourselves to the values $|\mu - 1| \ll 1$, we have from (4.4)

$$v_y = -\frac{B_y}{\sqrt{4\pi\rho}}\left\{1 + \frac{2\pi M}{B} - \frac{\pi^2}{2}\frac{M^2}{B^2}\left(\frac{3}{8} + \frac{B_x^2}{4B^2} + \frac{3}{8}\frac{B^2}{B_x B_y}\operatorname{arctg}\frac{B_y}{B_x}\right) + O\left(\frac{M^3}{B^3}\right)\right\} + \text{const.}$$

Equations (4.1) can be investigated in the case of an arbitrary magnetization law $\mu = \mu(\rho, H)$. Then the temperature in a simple wave is not altered, and the dependence of the transverse component of the velocity on the induction is determined by the second of Eqs. (4.1), whereby

$$\theta = \frac{B_x}{\sqrt{4\pi\rho}}\sqrt{\frac{\gamma}{\mu}}, \qquad \gamma = \frac{\mu^2 B + \mu_H B_x^2}{\mu^2 B + \mu_H B^2}.$$

Since

$$\theta\frac{d\theta}{dB_y} = -\frac{B_y B_x^2 \mu^2}{8\pi\rho\gamma^2(\mu + \mu_H H)B}\,\frac{3B_x^2\mu_H(\mu^2 + \mu_H B)^3 + 4\pi B_y^2\mu^4 M_{HH}}{(\mu^3 H + \mu_H B_x^2)^2}, \quad (4.5)$$

it follows from Eqs. (2.3) that $d\theta/dB_y > 0$ in a paramagnetic liquid and $d\theta/dB_y < 0$ in a diamagnetic liquid for waves propagating in the positive direction of the x axis.

The integral curves $\sqrt{\rho}v_y(B_y)$ in the $(B_y, \sqrt{\rho}v_y)$ plane intersect the $(\sqrt{\rho}v_y)$ axis at a constant angle $-$ arctan $(1/\sqrt{4\pi\rho\mu})$ and decrease monotonically as B_y increases. In a diamagnetic liquid these curves are convex downward, and in a paramagnetic liquid, upward.

It follows from (4.5) that the profile of a simple magnetosonic wave is deformed with the passage of time.

Differentiating (4.2) with respect to x with t constant and using the last of Eqs. (1.2), we obtain

$$-\frac{t\,d\theta/dB_y + dF/dB_y}{\theta(B_y)}\frac{dB_y}{dt} = 1.$$

From this we obtain that on the sections of the profile of a simple wave in a diamagnetic liquid where magnetization occurred ($dF/dB_y < 0$) at the initial instant of time, it is maintained at subsequent times. In the demagnetization section ($dF/dB_y > 0$) demagnetization occurs up until the time

$$t_1 = \min\left[-\frac{dF/dB_y}{d\theta/dB_y}\right].$$

At time $t = t_1$ an inflection point is formed on the demagnetization section of the profile of a simple wave [10]:

$$\left(\frac{\partial x}{\partial B_y}\right)_t = 0, \qquad \left(\frac{\partial^2 x}{\partial B_y^2}\right)_t = 0, \quad (4.6)$$

which indicates the onset of a demagnetization shock wave.

The system of equations (4.6) serves to determine the time and place of formation of the shock wave.

Similarly, it has been shown that the shock waves which arise in a paramagnetic liquid upon deformation of the profile of a simple wave are magnetization waves. In contrast to Alfvén waves, these shock waves of weak intensity have a time-independent structure [13].

The author expresses his gratitude to I. E. Tarapov for his constant attention to this research.

LITERATURE CITED

1. S. Vonsovskii, Magnetism [in Russian], Nauka, Moscow (1971).
2. I. Ya. Kagan, V. G. Rykov, and E. I. Yantovskii, "Ferromagnetic electrically conducting liquids," Magn. Gidrodinam., No. 3 (1970).

3. I. E. Tarapov, "The hydrodynamics of polarized and magnetized media," Magn. Gidrodinam., No. 1 (1972).
4. L. I. Sedov, The Mechanics of a Continuous Medium [in Russian], Vol. 1, Nauka, Moscow (1976).
5. N. E. Kochin, I. A. Kibel', and N. V. Roze, Theoretical Hydromechanics [in Russian], Part 2, GITTL, Moscow (1963).
6. I. E. Tarapov, "Sound waves in a magnetized medium," Zh. Prikl. Mekh. Tekh. Fiz., No. 1 (1973).
7. I. G. Shaposhnikov and M. I. Shliomis, "The hydrodynamics of magnetized media," Magn. Gidrodinam., No. 1 (1975).
8. N. F. Patsegon and I. E. Tarapov, "Sound and simple waves in a conducting magnetized medium," Ukr. Fiz. Zh., 19, No. 6 (1974).
9. A. G. Kulikovskii and G. A. Lyubimov, Magnetohydrodynamics [in Russian], GIFML, Moscow (1962).
10. A. I. Akhiezer (editor), Plasma Electrodynamics [in Russian], Nauka, Moscow (1974).
11. I. E. Tarapov, "Transverse waves and discontinuities in an ideal magnetized liquid," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 6 (1973).
12. N. F. Patsegon, R. V. Polovin, and I. E. Tarapov, "Simple waves and strong discontinuities in a magnetized medium," Prikl. Mat. Mekh., No. 1 (1979).
13. N. F. Patsegon, "The structure of a discontinuity of weak intensity in a conducting magnetized liquid," Magn. Gidrodinam., No. 2 (1978).

MAGNETOHYDRODYNAMICS OF HEAVY FLUIDS

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Four dimensionless parameters appear in the equations in connection with the discussion of the time-independent flow of an ideal compressible rotating plasma in a gravitational field: the Froude Fr, Rossby Ro, Mach M_0, and Alfvén A_0 numbers. Here it is assumed that A_0 and M_0 are simultaneously very small and satisfy the similarity relationship $A_0^2/M_0 = \nu_0$, where $\nu_0 = O(1)$ is a constant. First the case is analyzed in which $\mathrm{Fr} \to 0$ and $A_0^2/\mathrm{Fr}^2 = \lambda_0$, where $\lambda_0 = O(1)$ is a constant; the classical approximation of static equilibrium is obtained. If one notes that $\mathrm{Fr}^2 = \gamma M_0^2/\beta_0$, where β_0 is the ratio of characteristic lengths, then it is necessary to discuss two cases. The first case corresponds to $\beta_0 = O(1)$, and a limiting system of equations is derived which permits studying atmospheric motions near the planets of the solar system, for which the characteristic angular rotational velocity is not very high ($A_0^2/\mathrm{Ro} \ll 1$). The second case corresponds to $\beta_0 \to 0$ and $\beta_0/M_0 = \mu_0$, where $\mu_0 = O(1)$ is a new constant; it is possible to obtain a limiting system of equations which is suitable for analysis of the development of sunspots, where the magnetic and convective effects are closely linked.

1. Introduction

We will assume that only gravitational and electromagnetic forces are acting on the "fluid medium", which is treated as an ideal plasma (see [1] in connection with the definition of an ideal plasma). The equations which describe a nonsteady adiabatic flow of an infinitely conductive plasma rotating with angular velocity $\mathbf{\Omega}$ when viscosity and thermal conductivity are neglected have the form (the magnetic permeability μ is assumed to be constant):

$$\rho\{D\mathbf{v}/Dt + 2[\mathbf{\Omega}\cdot\mathbf{v}]\} + \nabla p + \rho g \mathbf{e}_3 = (1/\mu)[\mathrm{rot}\mathbf{B}\cdot\mathbf{B}]; \quad (1.1)$$

$$\partial\rho/\partial t + \mathrm{div}(\rho\mathbf{v}) = 0; \quad (1.2)$$

$$\mathrm{div}\,\mathbf{B} = 0; \quad (1.3)$$

$$\frac{DT}{Dt} - \frac{\gamma-1}{\gamma}\frac{T}{p}\frac{Dp}{Dt} = 0; \quad (1.4)$$

$$\partial\mathbf{B}/\partial t + \mathrm{rot}\,[\mathbf{B}\cdot\mathbf{v}] = 0. \quad (1.5)$$

The plasma is treated as an ideal gas with constant specific heats c_p and c_V ($\gamma = c_p/c_V$); therefore

Lille, France. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 2, pp. 37-42, March-April, 1980. Original article submitted March 20, 1979.

